- Holomorphic function

- $f(z)$ $z \in \mathbb{C}$ is holomorphic if:

$$f(z) = u(x,y) + i\, v(x,y)$$

dove $u(x,y) = \mathrm{Re}\{f(z)\}$

$v(x,y) = \mathrm{Im}\{f(z)\}$

- $U_x = \dfrac{\partial u(x,y)}{\partial x}$, $V_y = \dfrac{\partial v(x,y)}{\partial y}$

- $U_y = \dfrac{\partial u(x,y)}{\partial y}$, $V_x = \dfrac{\partial v(x,y)}{\partial x}$

$$\text{se: } \begin{cases} U_x = V_y \\ U_y = -V_x \end{cases} \Rightarrow f(z) \text{ is holomorphic}$$

~ es.: $z + 1 \rightarrow (x + 1) + i(y)$

$u = x + 1$; $v = y$

- $U_x = 1$ $V_y = 1$ $\rightarrow U_x = V_y$
- $U_y = \emptyset$ $V_y = \emptyset$ $\rightarrow U_y = -V_y$

$\rightarrow$ $z+1$ è olomorfe

- Propietà

- $\dfrac{h(z)}{g(z)}$ is holomorphic if
- $h(z)$ is holomorphic and $g(z)$ is holomorphic AND $g(z) \neq \emptyset \quad \forall z \in \Omega \subseteq \mathbb{C}$
  → NEED TO CHANGE the domain such that $g(z) \neq 0 \quad \forall z \in \underbrace{\Omega}_{\text{new domain}}$

- $h(z)$: holomorphic, $g(z)$ holomorphic
  → $h(z) + g(z)$: holomorphic
  → $h(z) \cdot g(z)$: holomorphic

- Propietà:

$f(z)$ is holomorphic if:

$$\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases} \qquad \begin{aligned} u_x &= \frac{\partial}{\partial x} f(x,y) \\ v_y &= \frac{\partial}{\partial y} f(x,y) \end{aligned}$$

(actually this defined as ENTIRE that is $f(z)$ is holomorphic in $\mathbb{C}$)

- Also we can rewrite the system as

$f(z)$ is holomorphic if $f_x + i f_y = \emptyset$

when: $\begin{cases} f_x = \frac{\partial}{\partial x} f(x,y) \\ f_y = \frac{\partial}{\partial y} f(x,y) \end{cases}$

@ Chapter 2