- Fourier Coefficients

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

$$a_m = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(k\omega_0 t)\,dt$$

$$b_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(k\omega_0 t)\,dt$$

such that $f(t)$ can be rewritten as:

$$f(t) = a_0 + \sum_{m=1}^{+\infty} a_m\cos(m\omega_0 t) + b_m\sin(m\omega_0 t)$$

~ Es.: $v(t) = \frac{V_M}{T} t \qquad t \in [0, T]$

$$\rightarrow a_v = \frac{1}{T} \int_{t_0}^{t_0+T} \frac{V_M}{T} \cdot t \, dt$$

$$= \frac{1}{2T} \frac{V_M}{T} \cdot \left[t^2 \right]_0^T = \frac{V_M}{2}$$

$$\rightarrow a_k = \frac{2}{T} \int_0^T \frac{V_M}{T} t \cdot \cos(k \omega_0 t) \, dt$$

$$= \frac{2V_M}{T^2} \cdot \left(\frac{1}{k^2 \omega_0^2} \cos(k\omega_0 t) + \frac{t}{k\omega_0} \sin(k\omega_0 t) \right)$$

$$= \frac{2V_M}{T^2} \left[\frac{1}{k^2 \omega_0^2} \cdot \cos(2\pi k - 1) \right]_0^T$$

$$= \emptyset \qquad \forall k \in \mathbb{N}$$

$$\rightarrow b_k = \frac{2}{T}\int_0^T \frac{V_M}{T} t \sin(k\omega_0 t)\,dt$$

$$= \frac{2V_M}{T^2}\int_0^T t\cdot \sin(k\omega_0 t)\,dt$$

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du \quad u = g(x)$$

$$= \frac{2V_M}{T^2 k\omega_0}\int_{k\omega_0(0)}^{k\omega_0(T)} \sin(u)\,du$$

$$= -\frac{2V_M}{T^2 k\omega_0}\Big[\cos(u)\Big]_0^{k\omega_0 T}$$

$$= -\frac{2V_M}{T^2 k\omega_0}\cdot\left(\cos(k\omega_0 T) - 1\right)$$

$$\omega_0 = \frac{2\pi}{T}$$

$$= -\frac{2V_M}{2\pi T\cdot k}\cdot\underbrace{\left(\cos(2\pi k) - 1\right)}_{\emptyset\ \forall k\in\mathbb{N}^+}$$