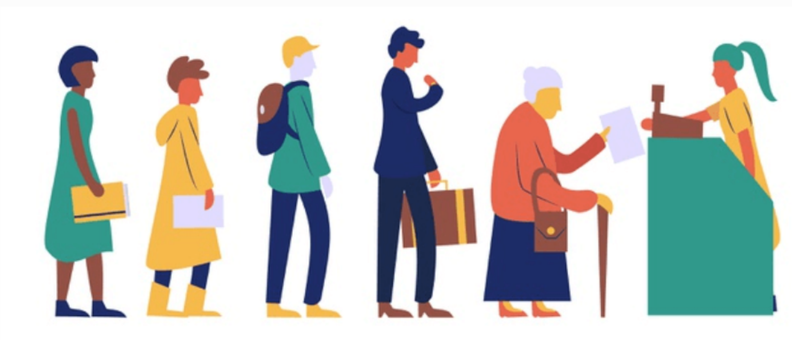
DEFINITION: A state automaton is a 5-tuple

$(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$, where:

- $\mathcal{E}$ is a discrete set of events
- $\mathcal{X}$ is a discrete set of states
- For each $x \in \mathcal{X}$, $\Gamma(x) \subseteq \mathcal{E}$ is the set of events that are possible in state x
- $f : \mathcal{X} \times \mathcal{E} \longrightarrow \mathcal{X}$ is a transition function

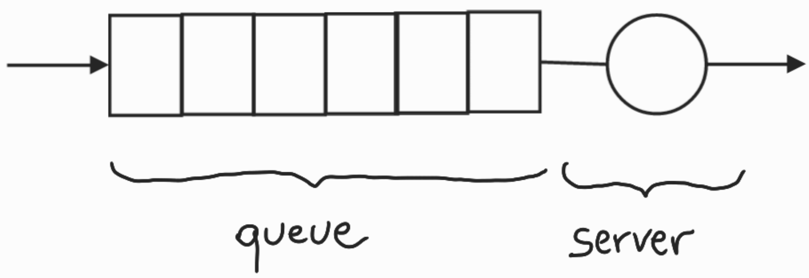
  $\rightsquigarrow$ For each $x \in \mathcal{X}$ and $e \in \Gamma(x)$, $x' = f(x, e)$ is the next state when the current state is x and the next event is e
- $x_0 \in \mathcal{X}$ is the initial state

EXAMPLE: Queueing system



$\leftsquigarrow$ bank, post-office, etc.

Represented as:



K: total capacity of the system

($K=7$ in the example above)

- 1 place in the server
- $K-1$ places in the queue

State automaton $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$:

- $\mathcal{E} = \{a, d\}$

  a: arrival of a customer

  d: termination of a service (in this case it coincides with a departure from the system)

- State $x = \#$ customers in the system

  ($x=5$ in the picture shown above)

  $\Rightarrow \mathcal{X} = \{0, 1, \dots, K-1, K\}$

- $\Gamma(0) = \{a\}$ ⇝ When there are no customers in the system, event d is not possible

$\Gamma(x) = \{a, d\}$ if $x > 0$

- $f(x,a) = \begin{cases} x+1 & \text{if } x < K \\ x & \text{if } x = K \end{cases}$

If the system is full, arrivals are rejected

$f(x,d) = x-1$ if $x > 0$

- $x_0 = 0$ (we assume the system initially empty)

We can also consider outputs in our models of discrete event systems:

<u>DEFINITION</u>: A state automaton with outputs is a 7-tuple $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0, \mathcal{Y}, g)$, where:

- $(\mathcal{E}, \mathcal{X}, \Gamma, f, x_0)$ is a state automaton
- $\mathcal{Y}$ is a discrete set of outputs
- $g : \mathcal{X} \to \mathcal{Y}$ is an output function