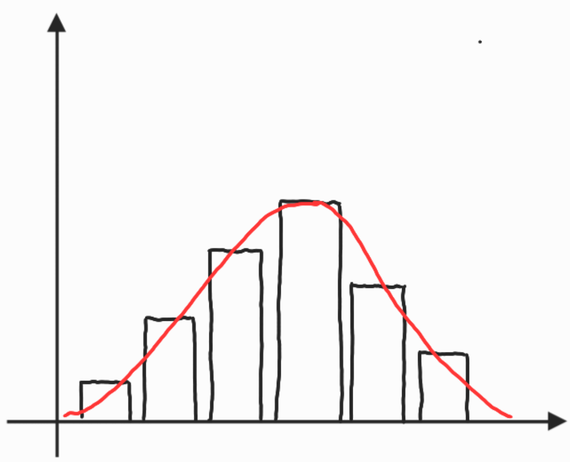
ESTIMATION OF MODELS OF DISCRETE EVENT SYSTEMS

- Assume that we would like to model a real DES using a CTHMC.

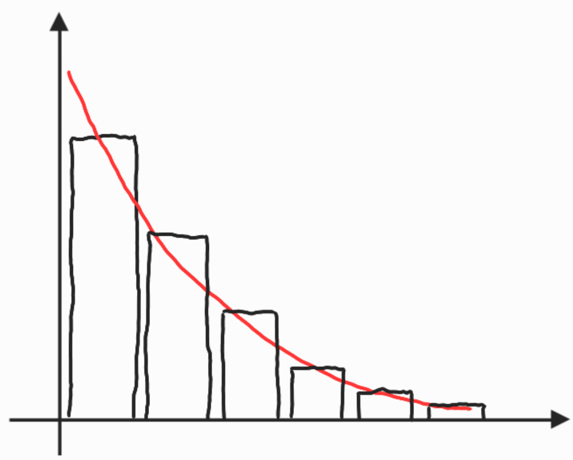
  Recall that state holding times in a CTHMC are exponentially distributed.

  Hence, a preliminary check may consist in the visual inspection of the histograms of the state holding times of the DES. If the histograms do not clearly resemble exponential pdfs, the modelling using a CTHMC should be abandoned.

  For state i of the DES, the histogram is built using recorded measurements $V(i)_1, V(i)_2, \ldots, V(i)_N$ of the state holding time.



=> It excludes modelling with a CTHMC



=> Compatible with a CTHMC

If we decide to proceed, we need to estimate the transition rate matrix Q from observations.

Recall that:

$$E[V(i)] = -\frac{1}{q_{i,i}}$$

It can be estimated as $\dfrac{V(i)_1 + V(i)_2 + \dots + V(i)_N}{N}$

$$\Rightarrow \boxed{\hat{q}_{i,i} = -\frac{N}{V(i)_1 + V(i)_2 + \dots + V(i)_N}}$$

Moreover,

$$p_{i,j} = -\frac{q_{i,j}}{q_{i,i}}$$

It can be estimated as $\dfrac{N_{i,j}}{N_i}$

Number of times that the system left state i to state j

Number of times that the system left state i

$$\Rightarrow \boxed{\hat{q}_{i,j} = -\frac{N_{i,j}}{N_i}\hat{q}_{i,i}}$$

- Stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, p_{X_0}, F)$

$\Rightarrow$ Estimate p, p_{X_0} and F from observations

- $p(j|i,e) = P(X_{k+1}=j \mid X_k = i, E_{k+1}=e)$

$$\Rightarrow \hat{p}(j|i,e) = \frac{N_{i,j}^{(e)}}{N_i^{(e)}}$$

$N_{i,j}^{(e)}$: Number of times that event e occurred in state i and the next state was j

$N_i^{(e)}$: Number of times that event e occurred in state i

- $p_{X_0}(i) = P(X_0 = i)$

$$\Rightarrow \hat{p}_{X_0}(i) = \frac{N_{\{X_0=i\}}}{N}$$

$N_{\{X_0=i\}}$: Number of times that the initial state was i

N: Number of times that the system was initialized

- $F = \{F_e : e \in \mathcal{E}\}$

$\Rightarrow$ $F_e \longleftarrow$ empirical cdf computed from measurements of the lifetimes

Summarizing:

$$\text{Real system} \xrightarrow[\text{Estimation}]{\text{measurements/observations}} \text{Model}$$