Proof (by construction)

For a stochastic timed automaton $(\mathcal{E}, \mathcal{X}, \Gamma, p, p_{x_0}, F)$ with Poisson clock structure, we have:

- $$V(i) \sim \text{Exp}\left(\frac{1}{\sum_{e \in \Gamma(i)} \lambda_e [1 - p(i|i,e)]}\right)$$

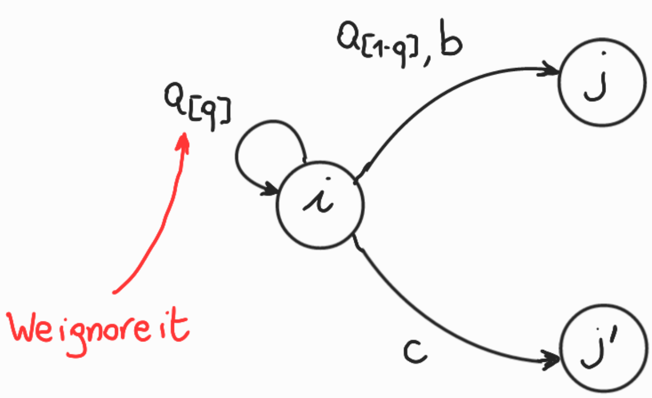
- $$p_{i,j} = \frac{\sum_{e \in \Gamma(i)} \lambda_e \, p(j|i,e)}{\sum_{j' \neq i} \sum_{e \in \Gamma(i)} \lambda_e \, p(j'|i,e)}$$

$$\sum_{j' \neq i} \sum_{e \in \Gamma(i)} \lambda_e \, p(j'|i,e) = \sum_{e \in \Gamma(i)} \lambda_e [1 - p(i|i,e)]$$

(see below)

Computed ignoring events over the loops (because they do not cause state transitions)

Example



$$\Rightarrow p_{i,j} = \frac{\lambda_a (1-q) + \lambda_b}{\lambda_a (1-q) + \lambda_b + \lambda_c}$$

$$\sum_{j' \neq i} \sum_{e \in \Gamma(i)} \lambda_e \, p(j'|i,e) = \sum_{e \in \Gamma(i)} \lambda_e \underbrace{\left[\sum_{j' \neq i} p(j'|i,e)\right]}_{1 - p(i|i,e)} = \sum_{e \in \Gamma(i)} \lambda_e [1 - p(i|i,e)]$$

For a CTHMC $(\mathcal{X}, Q, \pi_0)$ we have:

- $V(i) \sim Exp\left(-\frac{1}{q_{i,i}}\right)$
- $p_{i,j} = -\frac{q_{i,j}}{q_{i,i}}$

Hence, comparing the formulas for both models, we have:

$$-\frac{1}{q_{i,i}} = \frac{1}{\sum_{e \in \Gamma(i)} \lambda_e [1 - p(i|i,e)]}$$

$$-\frac{q_{i,j}}{q_{i,i}} = \frac{\sum_{e \in \Gamma(i)} \lambda_e \, p(j|i,e)}{\sum_{e \in \Gamma(i)} \lambda_e [1 - p(i|i,e)]}$$

From which we obtain the following relations:

$$\boxed{\begin{aligned} q_{i,i} &= -\sum_{e \in \Gamma(i)} \lambda_e [1 - p(i|i,e)] \\ q_{i,j} &= \sum_{e \in \Gamma(i)} \lambda_e \, p(j|i,e) \end{aligned}}$$

Moreover:

$$\pi_{0,i} = P(X(0) = i) = P(X_0 = 1) = p_{X_0}(i)$$

Hence:

$$\boxed{\pi_{0,i} = p_{X_0}(i)}$$

■