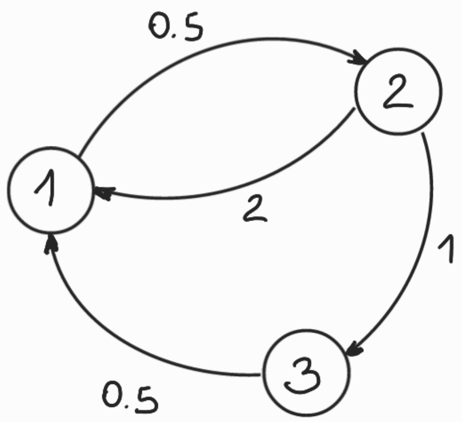
**COROLLARY**: The theorem above holds if $(\mathcal{X}, Q, \pi_0)$ is *irreducible* and *finite*.

## Example




Irreducible and finite

$\Downarrow$

We can apply the theorem

$$Q = \begin{bmatrix} -0.5 & 0.5 & 0 \\ 2 & -3 & 1 \\ 0.5 & 0 & -0.5 \end{bmatrix}$$

$$\begin{cases} \pi Q = 0 \\ \sum_{i=1}^{3} \pi_i = 0 \end{cases} \rightsquigarrow \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 & 0 \\ 2 & -3 & 1 \\ 0.5 & 0 & -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -0.5\pi_1 + 2\pi_2 + 0.5\pi_3 = 0 \\ 0.5\pi_1 - 3\pi_2 = 0 \\ \pi_2 - 0.5\pi_3 = 0 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

Notice that $1^{st}$ eq. $= -2^{nd}$ eq. $- 3^{rd}$ eq.

$\Rightarrow$ redundant

$\Rightarrow$ we can eliminate it

$$\Rightarrow \begin{cases} 0.5\pi_1 - 3\pi_2 = 0 & \Rightarrow \pi_1 = 6\pi_2 = 3\pi_3 \\ \pi_2 - 0.5\pi_3 = 0 & \Rightarrow \pi_2 = 0.5\pi_3 \\ \pi_1 + \pi_2 + \pi_3 = 1 & \end{cases}$$

$$\Rightarrow 3\pi_3 + 0.5\pi_3 + \pi_3 = 1 \Rightarrow \frac{9}{2}\pi_3 = 1 \Rightarrow \pi_3 = \frac{2}{9}$$

Finally, $\pi_1 = 3 \cdot \frac{2}{9} = \frac{2}{3}$ , $\pi_2 = \frac{1}{2} \cdot \frac{2}{9} = \frac{1}{9}$

$$\Rightarrow \pi = [\pi_1 \ \pi_2 \ \pi_3] = \left[\frac{2}{3} \ \frac{1}{9} \ \frac{2}{9}\right] \simeq [0.6667 \ 0.1111 \ 0.2222]$$

Using Matlab:

```
% Matrix Q
Q = [ -0.5 0.5 0 ; 2 -3 1 ; 0.5 0 -0.5 ];

% Limit probabilities (method 1, using the theorem)
n = 3; % Dimension of Q
pi = ([ Q' ; ones(1,n) ]\[ zeros(n,1) ; 1 ])'

% Limit probabilities (method 2, computing the limits numerically)
t = 1e6; % A big number (to simulate t --> inf)
pi0 = [ 1 0 0 ]; % An arbitrary one in this case (pi is independent of pi0)
pi = pi0*expm(Q*t)
```

Result (with both methods):

```
pi =

    0.6667    0.1111    0.2222
```