MAIN RESULT

For a stochastic timed automaton with Poisson clock structure, the residual lifetimes of an event have the same distribution as the corresponding total lifetimes.

In other words: $V_e \sim Exp\left(\frac{1}{\lambda_e}\right) \Rightarrow Y_e \sim Exp\left(\frac{1}{\lambda_e}\right)$

Sketch of the proof (by induction)

1) At initialization ($k=0$):

$Y_{e,0} = V_{e,1}$ for all $e \in \Gamma(X_0)$

Since $V_{e,1} \sim Exp\left(\frac{1}{\lambda_e}\right)$, then $Y_{e,0} \sim Exp\left(\frac{1}{\lambda_e}\right)$

$\Rightarrow$ The statement is true for $k=0$.

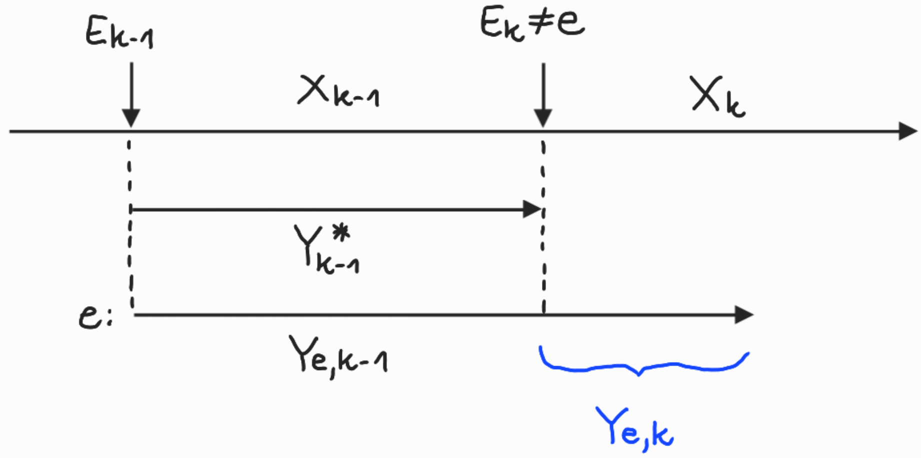
2) We assume that the statement is true for $k-1$, and we prove that it is true for k.

Case i) Event e is activated in state X_k.

$\Rightarrow Y_{e,k} = V_{e,N_{e,k}}$

Since $V_{e,N_{e,k}} \sim Exp\left(\frac{1}{\lambda_e}\right)$, then $Y_{e,k} \sim Exp\left(\frac{1}{\lambda_e}\right)$

Case ii) Event e "continues" in state X_k.



$$\Rightarrow \quad Y_{e,k} = Y_{e,k-1} - Y^*_{k-1}$$

$$P(Y_{e,k} \leq t \mid Y_{e,k-1} > Y^*_{k-1}) =$$

$$= P(Y_{e,k-1} - Y^*_{k-1} \leq t \mid Y_{e,k-1} > Y^*_{k-1})$$

$$= P(Y_{e,k-1} \leq t) = 1 - e^{-\lambda_e t}, \quad t \geq 0$$

Extended memoryless property

By induction

Hence, $Y_{e,k} \sim \mathrm{Exp}\left(\frac{1}{\lambda_e}\right)$.

$\Rightarrow$ The statement is true for k.

As a consequence of the main result, the following holds:

$$\boxed{P(E_k = e \mid X_{k-1} = x) = \frac{\lambda_e}{\Lambda(x)}}$$

where $\Lambda(x) \triangleq \sum_{e' \in \Gamma(x)} \lambda_{e'}$

We have a closed-form formula for $P(E_k = e \mid X_{k-1} = x)$!