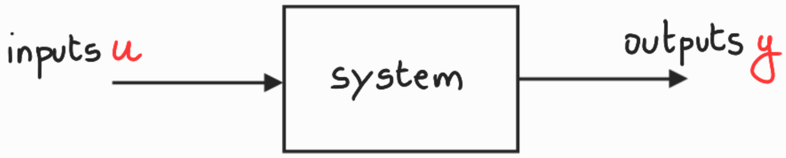
BASICS CONCEPTS OF SYSTEM THEORY

System: the part of the real world that is of interest for a particular study or application.



inputs: independent variables

- controllable
- uncontrollable (disturbances)

outputs: dependent variables

- controlled
- measured

Model: mathematical description of the relationship between system inputs and outputs.

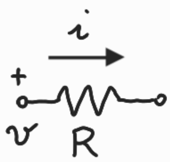
$$\boxed{y(\cdot) = F(u(\cdot))}$$

mapping between functions

The mapping F can be:

- explicit

  Example: model of a resistor



$v = Ri$

resistance

current (input)

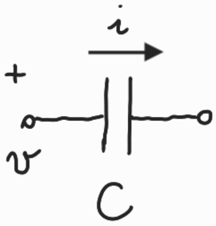
voltage (output)

- Ohm's Law -

If $i(t) = \cos(t)$, then $v(t) = R\cos(t)$.

- implicit (e.g. a differential equation)

<u>Example: model of a capacitor</u>



$$i(t) = \underbrace{C}_{\text{capacitance}} \frac{dv(t)}{dt}$$

If $i(t)=\cos(t)$, then we have to solve

$$\frac{dv(t)}{dt} = \frac{1}{C}\cos(t)$$

Integrating both sides:

$$\int_0^t dv(\tau) = \int_0^t \frac{1}{C}\cos(\tau)d\tau$$

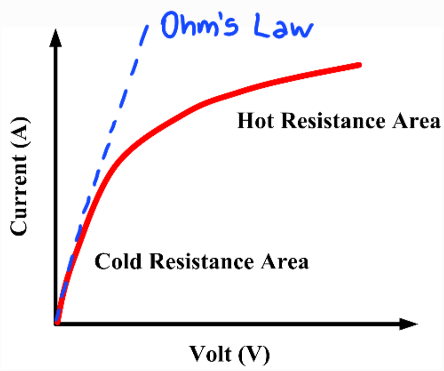
$$\Rightarrow v(t) - v(0) = \frac{1}{C}\Big[\sin(\tau)\Big]_0^t$$

$$\Rightarrow v(t) = \underbrace{v(0)}_{\substack{\text{value of the voltage}\\ \text{at time } t=0}} + \frac{1}{C}\sin(t)$$

<u>REMARKS</u>

- A model <u>is NOT</u> the system.

<u>Example: incandescent lamp modelled with Ohm's law</u>



$\Rightarrow$ Cope with uncertainty intrinsic in a model

- parametric uncertainties
- unmodelled behaviors

- The model of a system is <u>not unique</u>

  ↳ Different models for different applications and levels of accuracy or detail

How do we obtain the model of a system?

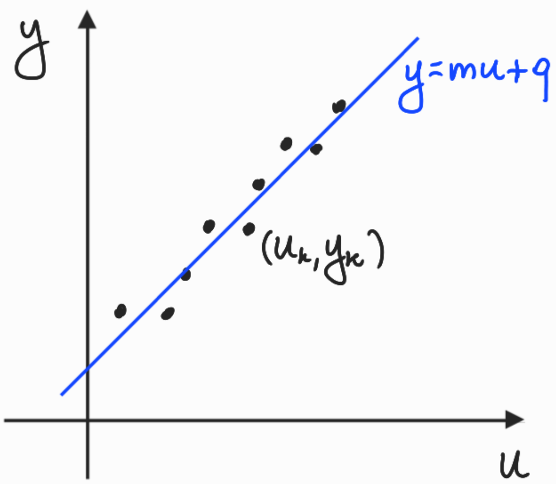
- Using first principles (e.g. laws of physics)

- Using measured data $\Rightarrow$ system identification
  - measure $u(\cdot)$ and the corresponding $y(\cdot)$
  - look for a mapping F minimizing the norm of the approximation error

$$\| y(\cdot) - F(u(\cdot)) \|$$

Example: least squares

Given the pairs $(u_1, y_1), (u_2, y_2), (u_3, y_3), \dots, (u_N, y_N)$,



find coefficients (m, q) that minimize

$$\underbrace{\frac{1}{N} \sum_{k=1}^{N} (y_k - m u_k - q)^2}_{\text{sum of squared errors}}$$